Electron EDM and soft leptogenesis in supersymmetric $B-L$ extension of the standard model

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Abstract

We analyze the connection between electric dipole moment of the electron and the soft leptogenesis in supersymmetric $B-L$ extension of the standard model. In this model, the $B-L$ symmetry is radiatively broken at TeV scale. Therefore, it is a natural framework for low scale seesaw mechanism and also for implementing the soft leptogenesis. We show that the phases of trilinear soft SUSY breaking couplings A, which are relevant for the lepton asymmetry, are not constrained by the present experimental bounds on electric dipole moment. As in the MSSM extended with right-handed neutrinos, successful leptogenesis requires small bilinear coupling B, which is now given by A_N and $B-L$ breaking VEVs. SUSY $B-L$ model with non-universal A-terms such that $A_N = 0$ while $A_\nu \neq 0$ is a promising scenario for soft leptogenesis. The proposed EDM experiments will test this scenario in the future.

1 Introduction

The current measurement of the baryon-to-entropy ratio of the Universe is given by [1]

$$Y_B \equiv \frac{n_B}{s} = (0.87 \pm 0.02) \times 10^{-10}, \tag{1}$$

where $s = 2\pi^2 g_\star T^3/45$ is the entropy density and $g_\star$ is the effective number of relativistic degrees of freedom. CP violation is an essential requirement in order to obtain this asymmetry. It is well known that in the standard model (SM), it is not possible to generate sufficient baryon asymmetry through the phase of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix, δ_{CKM} [2].

Supersymmetric (SUSY) extensions of the SM contain new CP-violating sources beyond δ_{CKM}, namely the Higgs bilinear term, μ, and the soft breaking terms (gaugino and squark soft masses, bilinear and trilinear couplings). The most stringent constraints on the SUSY phases come from continued efforts to measure the electric dipole moments (EDM) of the neutron, electron, and mercury atom [3].

Leptogenesis [4], based on a high scale seesaw mechanism, provides an attractive scenario to explain the baryon asymmetry. However, in this scenario, supersymmetry should be introduced to stabilize the electroweak scale. Therefore, leptogenesis is more natural in SUSY models. Recently, a new leptogenesis scenario, soft leptogenesis, has been proposed [5–7], where sneutrino decays offer new possibilities for generating the asymmetry.

Assuming universal soft SUSY breaking terms, the relevant terms for the soft leptogenesis in the minimal supersymmetric standard model (MSSM) extended with three right-handed neutrino superfields are given by

$$\mathcal{L}_{\text{soft}} = \frac{\tilde{m}_N^2}{2} \tilde{N}^{c\dagger} \tilde{N}^c + \frac{B_M^2}{2} \tilde{N}^c \tilde{N}^c + A_\nu Y_\nu \tilde{L} \tilde{N}^c H_2 + h.c., \tag{2}$$

and in the case of mSUGRA, $B_M^2 = B_N M_N$. This sector has one physical CP violating phase

$$\phi_\nu = \arg(A_\nu B_N^*). \tag{3}$$

In this respect, a mixing between the sneutrino $\tilde{N}^c$ and the anti-sneutrino $\tilde{N}^{c\dagger}$ is an analogue to the $B^0-\bar{B}^0$ and $K^0-\bar{K}^0$ systems. The mass difference and the two sneutrino mass eignestates are given by

$$\Delta M = |B_N|, \qquad \Delta\Gamma = \frac{2|A_\nu|\Gamma}{M_N}. \tag{4}$$

The CP violation in the $\tilde{N}^c$-mixing, induced by the phase ϕ_ν, generates lepton asymmetry in the final states of the $\tilde{N}^c$-decay. This lepton asymmetry is converted to baryon asymmetry through the sphaleron process [8]. The baryon to entropy ratio for $M_N \gg A_\nu$ case is given by [6]

$$\frac{n_B}{s} \simeq -10^{-3}\eta\left[\frac{4\Gamma|B_N|}{4|B_N|^2+\Gamma^2}\right]\frac{|A_\nu|}{M_N}\sin\phi_\nu, \tag{5}$$

where η is the efficiency parameter which is suppressed for small and large M_N because of the insufficient $\tilde{N}$ production and strong washout effect.

It has been noticed [6] that for 1 TeV $\ll M_N \leq 10^8$ soft leptogenesis may give important contribution to the baryon asymmetry only if the parameter B_N is very small. It means that, in this case, deviation from resonant condition gives too small baryon asymmetry.

The TeV scale right-handed neutrino is naturally obtained in supersymmeric $B-L$ extension of the standard model (SUSY $B-L$), which is based on the gauge group $G_{B-L} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. In this type of model, the $B-L$ Higgs potential receives large radiative corrections that induce spontaneous $B-L$ symmetry breaking at TeV scale, in analogy to the electroweak symmetry breaking in MSSM [9]. This result provides further motivation for considering the phenomenological and cosmological implications of this model.

In this paper, we investigate the possibility of soft leptogenesis in minimal SUSY $B-L$ model. This model has the B-term coming from a new A-term $A_N\tilde{N}\tilde{N}\chi_1$ and μ-term $\mu'\chi_1\chi_2$, where $\tilde{N}$ is sneutrino and $\chi_{1,2}$ are scalars which break $U(1)_{B-L}$ by their vacuum expectation values (VEV). We study the condition of the B-term for successful

soft leptogenesis in SUSY $B-L$ model, which derives relation between A_N, μ' and $B-L$ breaking VEVs. We also investigate the relation between electron EDM and soft leptogenesis, which both are generated by the same order one phase. Electron EDM and soft leptogenesis in the MSSM has been studied in Ref.[10], and the result is that contribution to electron EDM from soft SUSY breaking terms is well suppressed and soft leptogenesis works without constraints from EDMs. We show that this result holds in $B-L$ model as well, because of the small Dirac neutrino Yukawa couplings. However, planned future experiments will test our model.

The paper is organized as follows. In section 2 we discuss the Minimal SUSY $B-L$ which accounts for three right-handed neutrinos at TeV scale. In section 3 we study electron EDM from the CP violating phases in the (s)neutrino sector which is responsible for soft-leptogensis. The analysis of soft leptogenesis in this class of models is discussed in section 4. Finally we give our conclusions in section 5.

2 Supersymmetric $B-L$ extension of the SM

A low scale $B-L$ symmetry breaking, based on the gauge group $G_{B-L} \equiv SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, has been recently considered [11–13]. It was shown that this model can account for the current experimental results of the light neutrino masses and their large mixing. Therefore, it can be considered as one of the strong candidates for minimal extensions of the SM. Moreover, it was demonstrated that, similar to the electroweak symmetry breaking in MSSM, the $U(1)_{B-L}$ symmetry is radiatively broken at TeV scale in supersymmetric extension of this class of model [9]. Therefore, This type of models provides a natural framework for implementing TeV seesaw mechanism.

The part of SUSY $B-L$ superpotential, which is relevant for our analysis, is given by

$$W = Y_{\nu ij} N_i^c L_j H_2 - Y_{eij} E_i^c L_j H_1 + \frac{1}{2} Y_{Nij} N_i^c N_j^c \chi_1 + \mu H_1 H_2 + \mu' \chi_1 \chi_2, \tag{6}$$

where $i,j = 1\dots 3$ are generation indices and the superfields E^c, $L=(N,E)$, N^c contain

the leptons e_R^c, (ν_L, e_L), ν_R^c, respectively. $\chi_{1,2}$ are SM gauge singlet superfields which break $B-L$ symmetry by their VEVs. Note that Y_{B-L} for leptons and Higgs are given by $Y_{B-L}(L) = -Y_{B-L}(E^c) = -Y_{B-L}(N^c) = -1$, $Y_{B-L}(H_1) = Y_{B-L}(H_2) = 0$, $Y_{B-L}(\chi_1) = -2$, and $Y_{B-L}(\chi_2) = 2$.

The associated soft SUSY breaking terms (assuming that SUSY breaking scale is larger than $B-L$) are in general given by [9]

$$\begin{aligned}
-\mathcal{L}_{soft} &= \widetilde{m}^2_{Lij}\widetilde{L}_i^\dagger \widetilde{L}_j + \widetilde{m}^2_{Eij}\widetilde{E}_i^{c*}\widetilde{E}_j^c + \widetilde{m}^2_{Nij}\widetilde{N}_i^{c*}\widetilde{N}_j^c + m^2_{\chi_1}|\chi_1|^2 + m^2_{\chi_2}|\chi_2|^2 \\
&+ \Big[Y^A_{\nu ij}\widetilde{N}_i^c \widetilde{L}_j H_2 - Y^A_{eij}\widetilde{E}_i^c \widetilde{L}_j H_1 + \frac{1}{2} Y^A_{Nij}\widetilde{N}_i^c\widetilde{N}_j^c \chi_1 + B\mu' \chi_1\chi_2 \\
&+ \frac{1}{2}M_1 \widetilde{B}\widetilde{B} + \frac{1}{2}M_2 \widetilde{W}^a\widetilde{W}^a + \frac{1}{2}M_3 \widetilde{g}^a\widetilde{g}^a + \frac{1}{2}M_{B-L}\widetilde{Z}_{B-L}\widetilde{Z}_{B-L} + h.c \Big] .
\end{aligned} \tag{7}$$

Note that, due to the $B-L$ invariance, the bilinear coupling $B^2_{Nij}\widetilde{N}_i^c\widetilde{N}_j^c$ is not allowed. It may be generated only after the $B-L$ symmetry breaking by the vacuum expectation values $\langle \chi_{1,2} \rangle = v'_{1,2}$. In this case, B_N^2 is given by $B_N^2 = -v'_1 Y_N^A + Y_N v'_2 \mu'^*$.

The $B-L$ minimization conditions can be used to determine the supersymmetric parameter μ' [9]. Similar to the electroweak breaking condition in MSSM, one finds

$$\mu'^2 = \frac{m^2_{\chi_2} - m^2_{\chi_1}\tan^2\theta}{\tan^2\theta - 1} - \frac{1}{2}M^2_{Z_{B-L}}, \tag{8}$$

where $v'_1 = v'\sin\theta,\ v'_2 = v'\cos\theta$ and $U(1)_{B-L}$ gauge boson mass $M^2_{Z_{B-L}} = 8g^2_{B-L}v'^2$.

After imposing the electroweak and $B-L$ symmetry breaking conditions, one can compute the spectrum at low energy scale and analyze possible phenomenological consequences. Here, we present the general expressions for the charged slepton and sneutrino mass matrices in SUSY $B-L$. Now we adopt the super-MNS basis which, in analogue to the super-CKM basis in the quark sector, is defined as follows. Given the Yukawa matrices, we perform unitary transformations of the lepton superfields $L = (N, E)$, E^c and N^c such that the lepton mass matrices take diagonal forms:

$$\begin{aligned}
N_L &\rightarrow V_L^\nu N_L , \\
E_{L,R} &\rightarrow V^e_{L,R} E_{L,R} ,
\end{aligned} \tag{9}$$

with $Y^\nu_{eff} \to (V^\nu_L)^T Y^\nu_{eff} V^\nu_L = \mathrm{diag}(h_{\nu_e}, h_{\nu_\mu}, h_{\nu_\tau})$ and $Y^e \to (V^e_R)^\dagger Y^e V^e_L = \mathrm{diag}(h_e, h_\mu, h_\tau)$. In this basis, the leptonic charged current interactions is given by

$$-\frac{g}{\sqrt{2}}\left(\overline{\ell}_{Li}\gamma^\mu (V_L^{e\dagger} V_L^\nu)_{ij} \nu_{Lj} W_\mu + h.c.\right), \tag{10}$$

where g is the weak $SU(2)_L$ gauge coupling. The lepton flavour mixing matrix is then given by

$$U_{MNS} = V_L^{e\dagger} V_L^\nu. \tag{11}$$

We will assume diagonal charged lepton mass matrix, *i.e.*, $V^e_L = V^e_R = I$ and $V^\nu_L = U_{MNS}$.

In this class of models with TeV scale right-(s)neutino, the low-energy sneutrino mass matrix is more involved [9]. It turns out that the sneutrino is 12×12 hermitian matrix. In the basis of $(\phi_{\nu_L}, \phi_{\nu_R})$ with $\phi_{\nu_L} = (\tilde{\nu}_L, \tilde{\nu}^*_L)$ and $\phi_{\nu_R} = (\tilde{N}^c, \tilde{N}^{c*})$, it is given by [14]

$$\mathcal{M}^2 = \begin{pmatrix} M^2_{\nu_L\nu_L} & M^2_{\nu_L\nu_R} \\ M^2_{\nu_R\nu_L} & M^2_{\nu_R\nu_R} \end{pmatrix},$$

where $M^2_{\nu_A\nu_B}$ $(A, B \equiv L, R)$ can be written as [14]

$$M^2_{\nu_A\nu_B} = \begin{pmatrix} M^2_{A^\dagger B} & M^{2*}_{A^T B} \\ M^2_{A^T B} & M^{2*}_{A^\dagger B} \end{pmatrix},$$

with

$$\begin{aligned}
M^2_{\tilde{\nu}_L^\dagger \tilde{\nu}_L} &= U^\dagger_{MNS}\tilde{m}^2_L U_{MNS} + \frac{m_Z^2}{2}\cos 2\beta + v^2 \sin^2\beta U^\dagger_{MNS}(Y^\dagger_\nu Y_\nu)U_{MNS}, \\
M^2_{\tilde{\nu}_R^\dagger \tilde{\nu}_R} &= \tilde{m}^2_N + M^2_N + v^2\sin^2\beta(Y^*_\nu Y^T_\nu), \\
M^2_{\tilde{\nu}_L^T \tilde{\nu}_R} &= -v\sin\beta\, U^T_{MNS}.(Y^A_\nu)^T - v\cos\beta\mu U^T_{MNS}.(Y_\nu)^T\ , \\
M^2_{\tilde{\nu}_R^T \tilde{\nu}_R} &= M_N\mu'\cos\beta - v'\sin\theta\ Y^A_N\ , \\
M^2_{\tilde{\nu}_L^\dagger \tilde{\nu}_R} &= v\sin\beta U^\dagger_{MNS}(Y_\nu)^\dagger M_N\ , \\
M^2_{\tilde{\nu}_L^T \tilde{\nu}_L} &= 0,
\end{aligned} \tag{12}$$

where $M_N = Y_N v' \sin\theta$. Here few comments are in order: *i*) In the super-MNS basis, we do not perform any rotation by N^c_i since it is already assumed (without loss of generality)

that M_N is in diagonal form. *ii*) In the above expressions, we have kept the contribution to Eq.(12) proportional to the Dirac mass of the neutrinos because in general the unitary transformation which diagonalised Y^{ν}_{eff} doesn't necessarily diagonalise $Y^{\nu\dagger}Y^{\nu}$. *iii*) In general, the order of magnitude of the sneutrino mass matrix is as follows:

$$\mathcal{M}^2 \simeq \left(\begin{array}{cc} \mathcal{O}(v^2) & \mathcal{O}(vv') \\ \mathcal{O}(vv') & \mathcal{O}(v'^2) \end{array}\right). \tag{13}$$

Since $v' \sim$ TeV, the sneutrino matrix element are of the same order and there is no a seesaw type behavior as usually found in MSSM extended with heavy right-handed neutrinos. Therefore a significant mixing among the left- and right- handed sneutrinos is obtained. *iv*) The trilinear couplings A_ν and A_N are the only SUSY sources for CP violation in the sneutrino mass matrix (assuming μ and μ' are real). The impact of this feature on the electron EDM and soft leptogenesis will be analyzed in next sections. *v*) The results of light neutrino masses can be accommodated in this class of models with $M_N \sim \mathcal{O}(1)$ TeV if the neutrino Yukawa coupling Y_ν is of order $\lesssim 10^{-6}$ [9, 11, 12], which is close to the order of magnitude of the electron Yukawa coupling. Therefore, if one neglects the contributions proportional to Y_ν^2, the sneutrino mass matrix in the $(\tilde{\nu}_L, \tilde{\nu}_L^*, \tilde{N}^c, \tilde{N}^{c*})$ basis can be written as (where all the entries are 3×3 matrices):

$$\mathcal{M}^2 \simeq \left(\begin{array}{cccc} \tilde{m}_L^2 + \frac{1}{2}m_Z^2\cos 2\beta & 0 & v_2 Y_\nu^\dagger M_N & -v_2\; U^\dagger_{MNS}.(Y_\nu^A)^\dagger \\ 0 & \tilde{m}_L^2 + \frac{1}{2}m_Z^2\cos 2\beta & -v_2\; U^T_{MNS}.(Y_\nu^A)^T & v_2 Y_\nu^T M_N \\ v_2 Y_\nu M_N & -v_2 (Y_\nu^A)^* U^*_{MNS} & \tilde{m}_N^2 + M_N^2 & -v_1'\;(Y_N^A)^* + v_2' Y_N \mu' \\ -v_2 (Y_\nu^A) U_{MNS} & v_2 Y_\nu^* M_N & -v_1'\; Y_N^A + v_2'\; Y_N \mu'^* & \tilde{m}_N^2 + M_N^2 \end{array}\right). \tag{14}$$

It is worth noting that, in general, the trilinear coupling is not proportional to the corresponding Yukawa coupling. Therefore, Y_ν^A in general is not suppressed by the small Y_ν. Also, the Yukawa coupling Y_N is essentially unconstrained. In this case, the off-diagonal elements is expected to be of the same order as the diagonal ones. Thus, a large mixing is obtained and mass insertion approximation is not a proper approximation in this scenario.

The above sneutrino mass matrix can diagonalized by

$$\Gamma_{\tilde{\nu}}\mathcal{M}^2\Gamma_{\tilde{\nu}}^\dagger = \mathcal{M}_{diag}^2. \tag{15}$$

Hence,

$$(\tilde{\nu}_{phy})_i = (\Gamma_{\tilde{\nu}})_{ij}\tilde{\nu}_j \ , \qquad i,j = 1,2...,12. \tag{16}$$

For later convenience, we mention sneutrino sector for $v_{1,2} = 0$ case, because we will consider leptogenesis by sneutrino decay above the electroweak breaking scale.

For $v_{1,2} = 0$, off-diagonal blocks of Eq.(14) vanish. Thus, one can easily observe that 12×12 sneutrino mass matrix Eq.(14) is divided to two sperate mass matrices for the left-handed sneutrino with no-mixing and the right-handed sneutrino mass matrix with mixing of order v'. The 6×6 block part of the sneutrino mass matrix in the $(\tilde{N}^c, \tilde{N}^{c\dagger})$ basis is

$$\mathcal{M}_N^2 = \begin{pmatrix} \tilde{m}_N^2 + M_N^2 & B_M^{2*} \\ B_M^2 & \tilde{m}_N^2 + M_N^2 \end{pmatrix}, \tag{17}$$

where $B_M^2 = -v' \sin\theta Y_N A_N + v' \cos\theta Y_N \mu' = M_N(-A_N + \mu' \cot\theta) \equiv M_N B_N$ (μ' has been assumed to be real). Here, the mSUGRA relation $Y_N^A = Y_N A_N$ and $M_N = Y_N v' \sin\theta$ has been used. Now we assume the universal soft SUSY breaking terms for $\tilde{m}_N^2$ and A_N, and mass matrix of the heavy right-handed neutrinos M_N, then B_M^2, is already diagonal. One finds the mass eigenvalues of sneutrinos to be

$$M^2_{\tilde{N}_{\mp i}} = M_{Ni}^2 + \tilde{m}_{Ni}^2 \mp M_{Ni}\left|B_N\right|, \tag{18}$$

where

$$\left|B_N\right| = \sqrt{(-\left|A_N\right|\cos\theta_{A_N} + \mu'\cot\theta)^2 + \left|A_N\right|^2 \sin^2\theta_{A_N}}, \tag{19}$$

and the mass eigenstates

$$\tilde{N}_{+i} = \frac{1}{\sqrt{2}}\left(e^{i\phi/2}\tilde{N}_i^c + e^{-i\phi/2}\tilde{N}_i^{c\dagger}\right) \ , \tag{20}$$

$$\tilde{N}_{-i} = \frac{-i}{\sqrt{2}}\left(e^{i\phi/2}\tilde{N}_i^c - e^{-i\phi/2}\tilde{N}_i^{c\dagger}\right) \ , \tag{21}$$

for each generation $i = 1, 2, 3$. The phase of B_M^2, $\phi = \arg(B_M^2)$, is given by

$$\phi = \tan^{-1}\left[-\frac{|A_N|\sin\theta_{A_N}}{(-|A_N|\cos\theta_{A_N} + \mu'\cot\theta)}\right]. \tag{22}$$

On the other hand, the Higgs sector of this model consists of two Higgs doublets and two Higgs singlet with no mixing [9]. However, after the $B-L$ symmetry breaking, one of the four degrees of freedom contained in the two complex singlet χ_1 and χ_2 are swallowed in the usual way by the Z^0_{B-L} to become massive. Therefore, in addition to the usual five MSSM Higgs bosons: neutral pseudoscalar Higgs bosons A, two neutral scalars h and H and a charged Higgs boson $H^{\pm}$, the three new degrees of freedom remain physical [9]. They form a neutral pseudoscalar Higgs boson A' and two neutral scalars h' and H'. Their masses at tree level are given by

$$m^2_{A'} = \mu_1^2 + \mu_2^2, \tag{23}$$

$$m^2_{H',h'} = \frac{1}{2}\left(m^2_{A'} + M^2_{Z_{B-L}} \pm \sqrt{(m^2_{A'} + M^2_{Z_{B-L}})^2 - 4m^2_{A'}M^2_{Z_{B-L}}\cos^2 2\theta}\right). \tag{24}$$

Here $\mu_\alpha^2 = m^2_{\chi_\alpha} + \mu'^2$ with $\alpha = 1, 2$ [9].

The physical CP-even extra-Higgs bosons $H'_\alpha = (h', H')^T$ and CP-odd Higgs bosons $A'_\alpha = (G', A')^T$ are obtained from the rotation by orthogonal matrices $O_{R(I)}$:

$$H'_\alpha = (O_R)_{\alpha\beta}\,\mathrm{Re}\chi_\beta, \quad A'_\alpha = (O_I)_{\alpha\beta}\,\mathrm{Im}\chi_\beta, \tag{25}$$

where

$$O_R = \begin{pmatrix} \cos\alpha_R & -\sin\alpha_R \\ \sin\alpha_R & \cos\alpha_R \end{pmatrix}, \quad O_I = \begin{pmatrix} \sin\alpha_I & \cos\alpha_I \\ -\cos\alpha_I & \sin\alpha_I \end{pmatrix}, \tag{26}$$

where the mixing angle α_R is given by

$$\alpha_R = \frac{1}{2}\tan^{-1}\left[\tan 2\theta \frac{m^2_{A'} + M^2_{Z_{B-L}}}{m^2_{A'} - M^2_{Z_{B-L}}}\right]. \tag{27}$$

The effect of the right-sneutrino mixing on the lepton asymmetry can be determined from the sneutrino interaction Lagrangian, which contains couplings of both Dirac Yukawa

coupling Y^ν and new interaction Y_N. For Y_ν, these are given in the basis of $(\tilde{N}_{-i}, \tilde{N}_{+i})$ by

$$\begin{aligned} -\mathcal{L}_{Y^\nu} &= \frac{1}{\sqrt{2}} Y^\nu_{ij} e^{-i\phi/2} \tilde{N}_{+i} \left[\ell^j_L \tilde{H}_{2L} + \left(A_\nu + M_{Ni} e^{i\phi} \right) \tilde{\ell}^j H_2 + \mu^* H_1^\dagger \tilde{\ell}^j \right] \\ &+ \frac{i}{\sqrt{2}} Y^\nu_{ij} e^{-i\phi/2} \tilde{N}_{-i} \left[\ell^j_L \tilde{H}_{2L} + \left(A_\nu - M_{Ni} e^{i\phi} \right) \tilde{\ell}^j H_2 + \mu^* H_1^\dagger \tilde{\ell}^j \right] + c.c. \end{aligned} \tag{28}$$

Since Eq.(28) contains the complex parameter A_ν as well as ϕ defined in Eq.(22), these can generate CP violating phenomena. We study lepton EDMs induced by the phase of A_ν in the next section before discussing leptogenesis in section 4.

3 EDM constraint

The present limit of the EDM of charged leptons are [15]

$$d_e < 1.6 \times 10^{-27} e \text{ cm}, \tag{29}$$

$$d_\mu < 1.8 \times 10^{-19} e \text{ cm}. \tag{30}$$

This is expected to further improve in the near future to become [16]

$$d_e < 10^{-33} e \text{ cm}, \qquad d_\mu < 10^{-25} e \text{ cm}. \tag{31}$$

It is clear that the electron EDM provides the stringent constraint on any new CP violating contribution. Therefore, we will focus on the electron EDM constraint on the soft leptogenesis phase θ_{A_ν} and θ_{A_N}.

The effective Hamiltonian for the EDM of the electron can be written as [3]

$$H^{\text{EDM}}_{\text{eff}} = C_1 \mathcal{O}_1 + h.c., \tag{32}$$

where C_1 and $\mathcal{O}_1$ are the Wilson coefficient and the electric dipole moment operator respectively. The operator $\mathcal{O}_1$ is given by

$$\mathcal{O}_1 = -\frac{i}{2} \bar{e} \sigma_{\mu\nu} \gamma_5 e F^{\mu\nu}. \tag{33}$$

The supersymmetric contributions to the Wilson coefficient of the electron result from the one loop penguin diagrams with neutralino and chargino exchange. In the neutralino contribution the selectrons are running in the loop. While the chargino diagram involves the sneutrinos. As advocated in the previous section, the selectron mass matrix has no dependence on the CP violating phases of SUSY breaking terms associated with the neutrino: A_ν and A_N which give contribution to the soft leptogenesis. In this respect, the neutralino contribution to the electron EDM is not relevant for our analysis and we will focus here on the chargino contribution only. In this case we have

$$d_e/e = \mathrm{Im}\left(C_e^{\chi^+}\right), \tag{34}$$

where e is the electron electric charge. To compute the Wilson coefficient $C_e^{\chi^+}$ and study its dependence on the phase θ_{A_ν,A_N}.

The chargino interactions with lepton and sneutrino are given by

$$\begin{aligned}\mathcal{L}_{e\tilde{\nu}\chi^+} = g\sum_{k=1}^{2}\sum_{\alpha=1}^{12}\sum_{a=1}^{3} \Big(& - \; V_{k1}(U_{MNS})_{ab}\; \bar{e}_L^a\; (\chi_k^+)^* \; \sum_{b=1}^{3}(\Gamma_{\tilde{\nu}}^\dagger)_{b\alpha}\tilde{\nu}_{phy}^\alpha \\ & + \; U_{k2}^* \; [Y_e^{\mathrm{diag}}.U_{MNS}]_{ab}\; \bar{e}_R^a\; (\chi_k^+)^* \; \sum_{b=1}^{3}(\Gamma_{\tilde{\nu}}^\dagger)_{b\alpha}\tilde{\nu}_{phy}^\alpha \\ & - \; V_{k2}\,(Y_\nu)_{ab}\, \bar{e}_L^a(\chi_k^+)^* \sum_{b=7}^{9}(\Gamma_{\tilde{\nu}}^\dagger)_{b\alpha}\tilde{\nu}_{phy}^\alpha \Big).\end{aligned} \tag{35}$$

The above lagrangian matches the general form of the interaction due to the exchange of spinor ψ_i and scalar ϕ_k

$$\mathcal{L} = L_{ik}\bar{e}_R\psi_i\phi_k + R_{ik}\bar{e}_L\psi_i\phi_k + h.c. \tag{36}$$

In this case, one finds that EDM is given by [17]

$$d_e/e = \frac{m_i}{16\pi^2 m_k^2}\mathrm{Im}\left(L_{ik}R_{ik}^*\right)\left[Q_i A\left(\frac{m_i^2}{m_k^2}\right) + Q_k B\left(\frac{m_i^2}{m_k^2}\right)\right], \tag{37}$$

where the loop functions A and B are given by

$$A(x) \;=\; \frac{1}{2(1-x)^2}\left[3 - x + \frac{2\ln x}{1-x}\right], \tag{38}$$

$$B(x) \;=\; \frac{1}{2(1-x)^2}\left[1 + x + \frac{2x\ln x}{1-x}\right]. \tag{39}$$

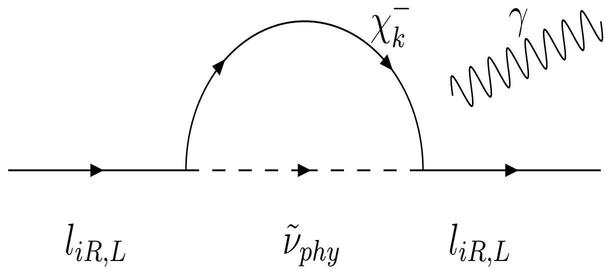

Figure 1: The chargino contributions to the charged lepton EDM due to chargino and sneutrino exchange.

The SUSY contribution to d_e due the exchange of χ_i and $\tilde{\nu}_k$ is shown in Fig.1. From Eqs.(35,36), one can identify L and R coefficients as

$$
\begin{aligned}
L_{k\alpha} &= h_e U_{k2}^* \,(U_{MNS})_{1b}(\Gamma_{\tilde{\nu}}^\dagger)_{b\alpha}, && (40)\\
R_{k\alpha} &= -g V_{k1}(U_{MNS})_{1b}\,(\Gamma_{\tilde{\nu}}^\dagger)_{b\alpha} - V_{k2}(Y_\nu)_{1b}(\Gamma_{\tilde{\nu}}^\dagger)_{(b+6)\alpha}. && (41)
\end{aligned}
$$

Therefore, the electron EDM is given by

$$
d_e/e = \sum_{k=1}^{2}\sum_{\alpha=1}^{12} Q_{\chi_k}\frac{m_{\chi_k}}{16\pi^2 m_{\tilde{\nu}_\alpha^2}}\mathrm{Im}\,(L_{k\alpha}R_{k\alpha}^*)\,A\left(\frac{m_{\chi_k^2}}{m_{\tilde{\nu}_\alpha}^2}\right). \qquad (42)
$$

Here we assume that phase in $\mathcal{M}^2$ of Eq.(14), that is, phase in the diagonalization matrix $\Gamma_{\tilde{\nu}}$, say $\sin\phi_\Gamma$, is the only origin of the contribution to the EDM. However, from Eqs.(40), (41) and (42), the combination $\mathrm{Im}(LR^*)$ vanishes for the first term of $R_{k\alpha}$ because $\Gamma_{\tilde{\nu}}$ dependence of $R_{k\alpha}$ is the same as that of $L_{k\alpha}$. The second term of $R_{k\alpha}$ gives non-zero contribution to electron EDM, but it is small because of suppression by Dirac neutrino Yukawa coupling $Y_\nu \sim 10^{-6}$. To estimate its magnitude, we assume $(\Gamma_{\tilde{\nu}}^\dagger)_{b\alpha}(\Gamma_{\tilde{\nu}})_{\alpha(b+6)} \sim e^{i\phi_\Gamma}$, $U_{MNS}, U, V \sim 1$ and $m_\chi = m_{\tilde{\nu}} = 100\mathrm{GeV}(1\mathrm{TeV})$, the eEDM is

$$
d_e/e \sim 10^{-31(32)}\sin\phi_\Gamma \quad \mathrm{cm}, \qquad (43)
$$

which is four (five) orders of magnitude smaller than the present experimental limit. Therefore eEDM does not constrain the phase $\sin\phi_\Gamma$ today, that is, $\theta_{A_\nu(A_N)}$, which is essential for soft leptogenesis as we will show in the next section. However the planned eEDM experiment Eq.(31) will give non-trivial tests of our scenario in the future.

4 Soft leptogenesis in SUSY $B-L$ model

In SUSY $B-L$ extension of the SM, the neutrino Yukawa coupling Y_ν is very tiny, therefore the standard thermal leptogenesis can not account for baryon asymmetry in the Universe unless a highly degenerate right-handed neutrino masses are assumed [12, 18].

Recently, a new source of lepton asymmetry, due to the induced mixing between sneutrino-antisneutrino, has been analyzed [5–7]. In this framework, the CP asymmetry in decay of the heavy sneutrino $\tilde{N}_- \equiv \tilde{N}_{-1}$ defined in Eq.(20) and (21) is given by

$$\begin{aligned}\epsilon_- &= \frac{\sum_f \left[\Gamma(\tilde{N}_- \to f) - \Gamma(\tilde{N}_- \to \bar{f})\right]}{\sum_f \left[\Gamma(\tilde{N}_- \to f) + \Gamma(\tilde{N}_- \to \bar{f})\right]} \\ &= \frac{2(M_-^2 - M_+^2)\Pi_{+-} \sum_f \mathrm{Im}(f_-^* f_+) c_f}{\sum_f \left[|f_+|^2 (M_-^2 - M_+^2)^2 + |f_- \Pi_{++} - f_+ \Pi_{+-}|^2\right]},\end{aligned} \tag{44}$$

where f is a final state with lepton number equal to 1 and $\bar{f}$ is its conjugate. In the second equation, $f_\pm$ are tree-level decay amplitudes and $\Pi_{\pm\pm,\pm\mp}$ are the absorptive part of two point functions, which are given below. The factor $c_f (f = B, F$ for bosonic and fermionic final state) is introduced to parametrize the phase space of the bosonic and fermionic final states. Asymmetry by $\tilde{N}_+$ is obtained by exchanging $+ \leftrightarrow -$ in Eq.(44). The effect of the $\tilde{N}^c - \tilde{N}^{c\dagger}$ mixing on the lepton asymmetry ϵ, which is assumed to be dominated the direct CP violation in this decay, can be determined by computing the sneutrino mass eigenstates.

4.1 MSSM+N_1 case

First we briefly mention the MSSM+heavy right handed (s)neutrino $N_1(\tilde{N}_1)$ case with $M_N \gg$ TeV [6]. In this model, CP asymmetry from heavy sneutrino $\tilde{N}_\pm$ decay into $\ell\tilde{H}$ and $\tilde{\ell}H$ is given by

$$\epsilon = \frac{4\Gamma B_N}{4B_N^2 + \Gamma^2} \frac{\mathrm{Im} A_\nu}{M_N} \Delta_{BF}, \tag{45}$$

where the total decay rate Γ is

$$\Gamma = \frac{(Y_\nu Y_\nu^\dagger)_{11}}{4\pi} M_N, \tag{46}$$

and $\Delta_{BF} \equiv (c_B - c_F)/(c_B + c_F)$. This gives the largest value of ϵ at $\Gamma = 2B_N$, which is the resonance condition, and therefore

$$M_N = \left(\frac{10^{-3}\text{eV}}{m_\nu}\right)^{1/2} \left(\frac{B_N}{100\text{GeV}}\right)^{1/2} 10^{10}\ \text{GeV} < \frac{\text{Im}A_\nu}{1\text{TeV}}\ 10^{8-9}\text{GeV}. \quad (47)$$

The equality comes from the resonance condition, and the inequality from that ϵ is large enough to obtain observed baryon asymmetry. One can see that this requires small b-term: $B_N \sim \mathcal{O}(10)$GeV.

4.2 $B - L$ case

Next we discuss our $U(1)_{B-L}$ model. It is worth mentioning that the leptogenessis process takes place at large scale, around $B-L$ breaking scale (v'). At this scale, the electroweak symmetry is still an exact symmetry, *i.e.*, $v = 0$. Thus, as given in section 2, one can easily observe that 12×12 sneutrino mass matrix Eq.(14) is divided to two sperate mass matrix for left-handed sneutrino with no-mixing and right-handed sneutrino mass matrix with mixing of order v'. We will focus on the lightest right-sneutrino $\tilde{N}_1$, and therefore its mass eigenstate $\tilde{N}_\pm$, since the lepton asymmetry is usually dominated by the decay of the lightest one.

We consider soft leptogenesis in this model. From Eq.(28), CP asymmetry is generated by decay processes of the lightest heavy sneutrino $\tilde{N}_\pm$ into $\tilde{\ell} + H$ and $\ell + \tilde{H}$. Moreover, if heavy neutrinos N_i and new particles H'_α, A'_α and $\chi_{1,2}$ are lighter than $\tilde{N}_\pm$, there are other decay modes: $\tilde{N}_\pm \to N_i + \chi_1$ and $\tilde{N}_+ \to \tilde{N}_- + (H', A')$. However, if the new particles are heavier than $\tilde{N}_\pm$, these decay modes are kinematically forbidden and can not contribute to CP asymmetry. In the following analysis, we neglect these new processes by assuming these new particles are heavier than $\tilde{N}_\pm$.

In this case, the total decay width of $\tilde{N}_-$ is given by

$$\Gamma_- = 2\Gamma(\tilde{N}_- \to \ell + \tilde{H}_2) + 2\Gamma(\tilde{N}_- \to \tilde{\ell} + H_2) + 2\Gamma(\tilde{N}_- \to \tilde{\ell} + H_1), \quad (48)$$

where the factor 2 comes from decay into anti-particles. The tree level contribution to

the decay of $\tilde{N}_-$ to Higgs doublet and charged lepton doublet is given by

$$
\begin{aligned}
\Gamma(\tilde{N}_- \to \ell + \tilde{H}_2) &= \frac{1}{2M_-} \sum_i \left| f_-(\ell^i \tilde{H}_2) \right|^2 I_2(M_-; m_{\ell^i}, m_{\tilde{H}_2}), &(49)\\
\Gamma(\tilde{N}_- \to \tilde{\ell} + H_2) &= \frac{1}{2M_-} \sum_i \left| f_-(\tilde{\ell}^i H_2) \right|^2 I_2(M_-; m_{\tilde{\ell}^i}, m_{H_2}), &(50)\\
\Gamma(\tilde{N}_- \to \tilde{\ell} + H_1) &= \frac{1}{2M_-} \sum_i \left| f_-(\tilde{\ell}^i H_1) \right|^2 I_2(M_-; m_{\tilde{\ell}^i}, m_{H_1}), &(51)
\end{aligned}
$$

where phase space integral I_2 is

$$
I_2(x; y, z) = \frac{1}{8\pi x^2} \sqrt{[x^2 - (y-z)^2]\,[x^2 - (y+z)^2]}, \tag{52}
$$

and $- \to +$ for $\tilde{N}_+$ decay. Tree-level amplitudes $f_\pm$ are defined as

$$
\begin{aligned}
f_\pm(\ell^i \tilde{H}_2) &= -iY^\nu_{1i} e^{i\phi/2} \sqrt{M_\pm^2 - m_{\tilde{H}_2}^2 - m_{\ell^i}}, &(53)\\
f_\pm(\tilde{\ell}^i H_2) &= -iY^\nu_{1i} e^{i\phi/2} \left(A_\nu^* \pm M_N e^{-i\phi} \right), &(54)\\
f_\pm(\tilde{\ell}^i H_1) &= -iY^\nu_{1i} e^{i\phi/2} \mu. &(55)
\end{aligned}
$$

In order to satisfy the out of equilibrium condition, we should have $\Gamma < H(M_-)$, where $H(M_-)$ is the Hubble parameter at temperature $T = M_-$, namely

$$
H(M_-) \simeq \frac{g_*^{1/2} M_-^2}{M_{pl}} \ . \tag{56}
$$

If M_- is of order 10^3 GeV (*i.e.*, it is dominated by soft scalar mass $\tilde{m}_N$, which can be of that order), then $H(M_-) \simeq 10^{-12}$ GeV, hence $\Gamma \lesssim 10^{-12}$ GeV is required. However for $M_N \simeq 10^3$ GeV and $Y_\nu \sim 10^{-6}$, the ratio $\Gamma/H \sim 10$, hence efficiency factor can be estimated to be $\eta \sim 0.1$. Then, baryon asymmetry is given by

$$
Y_B = -8.6 \times 10^{-4} \epsilon\ \eta. \tag{57}
$$

The CP asymmetry ϵ is generated by interference between the tree and the following self-energy diagram (Fig.2) with a bosonic loop. Diagrams of fermionic loops vanishes.

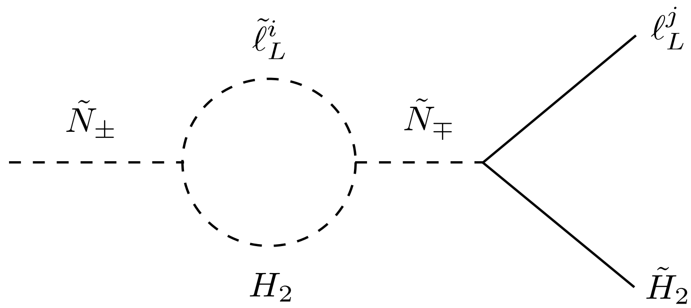

Figure 2: The contributions to the decay $\tilde{N}_- \to \ell^j_L \tilde{H}_2$. The bosonic final states $\tilde{\ell}^j_L H_2$ and $\tilde{\ell}^j_L H_1$ are also possible.

The two-point functions Π obtained from the diagram Fig.2 are given by

$$\Pi_{++} = M_+ \Gamma_+, \tag{58}$$

$$\Pi_{--} = M_- \Gamma_-, \tag{59}$$

$$\Pi_{+-} = 2|Y^\nu_{1i}|^2 |A_\nu|^2 M_N \sin(\phi - \theta_{A_\nu}) I_2(M_+; m_{\tilde{\ell}^i}, m_{H_2}), \tag{60}$$

$$\Pi_{-+} = 2|Y^\nu_{1i}|^2 |A_\nu|^2 M_N \sin(\phi - \theta_{A_\nu}) I_2(M_-; m_{\tilde{\ell}^i}, m_{H_2}). \tag{61}$$

The masses of sneutrinos $\tilde{N}_\pm$ must be strongly degenerate in order to obtain enough baryon asymmetry. Since the mass difference of $\tilde{N}_\pm$ is $M_+^2 - M_-^2 = M_N\,|B_N|$ from Eq.(18), the resonance condition becomes

$$M_N\,|B_N| \sim \Pi_{\pm\pm} \sim 10^{-8}\mathrm{GeV}^2. \tag{62}$$

This implies that $|B_N|$ has to be extremely small, $\sim 10^{-11}$ GeV, in order to satisfy the resonance condition. This means that more strict degeneracy between the heavy sneutrino masses is required in the $B-L$ model comparing with the MSSM+N_1 case. Although the resonance condition $\Gamma \sim B_N$ itself is the same, the value of Γ is different. One can see that $\Gamma_{MSSM+N} \gg \Gamma_{B-L}$ from Eqs.(46) and (49),(50),(51) because $M_N \sim 1$ TeV in the $B-L$ model.

From the definition of B_N, Eq.(19), parameters A_N, μ' and $\cot\theta = v_2'/v_1'$ have to be related to each other to satisfy the resonance condition Eq.(62); $|A_N| \cos\theta_{A_N} \simeq \mu' \cot\theta$

for the first term of B_N, and $\theta_{A_N} \ll 1$ and/or $|A_N| \ll \mathcal{O}(\mathrm{TeV})$ for the second term. While there are two possibilities for the second term, the condition for the first term is the same and this gives constraint on $\cot\theta$.

1. $\sin\theta_{A_N} \ll 1,\ |A_N| \sim \mathcal{O}(\mathrm{TeV})$ case;

   In this case, we obtain resonance condition for a new parameter ϵ_θ which parametrizes the deviation of $\cot\theta$ from $|A_N|\cos\theta_{A_N}/\mu'$ defined as

$$\epsilon_\theta \equiv 1 - \frac{|A_N|\cos\theta_{A_N}}{\mu'\cot\theta} \ll 1. \tag{63}$$

   The left panel of Fig.3 shows the baryon asymmetry as a function of ϵ_θ for $|A_N| = 10^3$ GeV (solid) and 10^2 GeV (dashed), assuming $\theta_{A_N} = 0$ and $|A_\nu| = 10^3$ GeV.

2. $|A_N| \ll \mathcal{O}(\mathrm{TeV})$ case;

   For the small $|A_N|$ case, θ_{A_N} dependence is not important and $\theta \simeq \pi/2$ is required when μ' is large ($\gtrsim 10^2$ GeV). The resonance condition for a new parameter x which parametrizes the deviation of θ from $\pi/2$ is

$$x \equiv \frac{\pi}{2} - \theta \ll 1. \tag{64}$$

   The right panel of Fig.3 shows the baryon asymmetry as a function of x for $|A_\nu| = 10^3$ GeV (solid) and 10^2 GeV (dashed), assuming $|A_N| = 0$.

From these figures one can see that enough baryon asymmetry is generated when $\epsilon_\theta \lesssim 10^{-7}$ or $x \lesssim 10^{-8}$. Namely, the resonance condition for SUSY $B-L$ model,

$$\frac{v_2'}{v_1'} = \frac{|A_N|}{\mu'}, \tag{65}$$

must be accurately satisfied for both $\theta_{A_N} \ll 1$ and/or $|A_N| \ll \mathcal{O}(\mathrm{TeV})$ cases. It is hard to realize the condition Eq.(65) for general A_N and μ'. However, there are several SUSY breaking scenarios yielding non-universal A-terms, and it is quite plausible to find a SUSY models with $|A_N| = 0$ and $A_\nu \neq 0$ [19]. In fact, in the modulus-dominated SUSY breaking case, A-terms are obtained as

$$A_{ijk} = -\sqrt{3}m_{3/2}(3 + n_i + n_j + n_k), \tag{66}$$

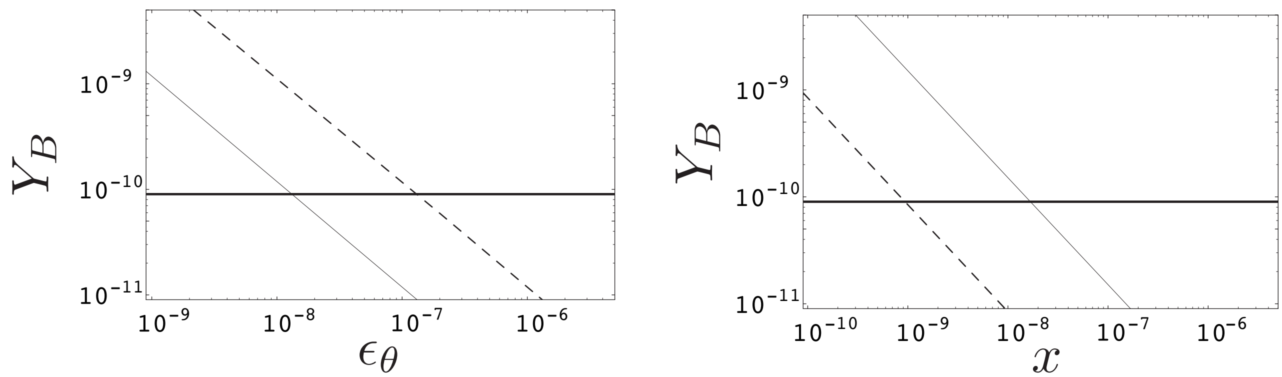

Figure 3: Baryon asymmetry as a function of new parameters ϵ_θ (left) and x (right) defined in Eqs.(63) and (64). In both figures, $M_N = 1$ TeV, $\mu' = 500$ GeV and $\theta_{A_\nu} = \pi/2$. The horizontal line is the experimental value Eq.(1).

where $m_{3/2}$ is the gravitino mass and n_i, n_j and n_k are modular weight of the fields to couple. Since the A-terms depend on the fields, these are in general non-universal. If we assign modular weight $n_i = -1$ for the heavy sneutrinos $\tilde{N}$ and singlet scalar χ_1, then $A_N = 0$. On the other hand, $n = -2$ for $\tilde{L}$ and $n = -3$ for H_2 give $A_\nu = 3\sqrt{3} m_{3/2} e^{-i\alpha}$, where α is the corresponding CP violating phase. The detailed phenomenological implications of this class of models have been analyzed in Ref.[19]. This type of model is a promising scenario for implementing soft leptogenesis.

5 Conclusions

We have studied electron EDM and soft leptogenesis induced by trilinear soft SUSY breaking terms in a $B-L$ extension of supersymmetric standard model. The $B-L$ symmetry is broken by VEVs of extra Higgs bosons at TeV scale and neutrino Dirac Yukawa coupling is of order 10^{-6} in this TeV scale seesaw model. Because of the smallness of Dirac neutrino Yukawa couplings, the electron EDM from higgsino loop is enough suppressed, and not impose constraint on the CP violating phases of the trilinear terms $A_{\nu,N}$. The soft leptogenesis is also generated by the same trilinear terms. Since the decay rate Γ of heavy sneutrino depends on the seesaw scale, the resonance condition $\Gamma \sim B_N$

requires small bilinear term $|B_N| \sim 10^{-11}$ GeV for TeV scale seesaw model. This resonance condition leads to relation between $\cot\theta$, μ' and A_N. SUSY model of non-universal A-terms such that $A_N = 0$ while $A_\nu \neq 0$ with large $\tan\theta$ is a promising scenario for successful soft leptogenesis. We have shown that this can be naturally realized in models with non-universal soft SUSY breaking terms, as in, for example, orbifold string models with an appropriate assignment of modular weight for each field. We have emphasized that this scenario has also testable implications at future collider. The future experiments of electron EDM will provide a serious test for the soft-letpgenesis in this class of SUSY models.

Acknowledgments

The authors would like to thank J. Ellis for useful discussions. This work is supported by the ESF grant No. 8090 (Y.K. and M.R.). The work of S.K. is supported in part by ICTP project 30 and the Egyptian Academy of Scientific Research and Technology.

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